My research plans (07/28/2010)

I.

There is no agreement on the nature of high temperature super conductivity (HTSC); for example see Philip W. Anderson's paper "Is There Glue in Cuprate Superconductors?" (Science, Vol 317, 22 June 2007).

It is clear that treatment of electrons as almost free ones does not work.

It might be worthwhile to turn to a deeper study of properties of electrons in a strong periodic potential.

For two electrons in a strong periodic potential Schrödinger equation is ($\vec{p} = -i\nabla$; here and below all the constants are set to unity)

$$H\Psi = \left\{ \sum_{\alpha=1}^{2} \frac{\vec{p}_\alpha^{\,2}}{2} + \sum_{\alpha=1}^{2} U(\vec{r}_\alpha) + V(|\vec{r}_1 - \vec{r}_2|) \right\} \Psi = E\Psi$$

If Coulomb repulsion were small, the solution could be sought as a perturbation series: $E = E^{(0)} + E^{(1)} + E^{(2)} + \ldots$

$$\Psi = \Psi^{(0)} + \Psi^{(1)} + \Psi^{(2)} + \ldots \qquad H_0\Psi^{(0)} = \left\{ \sum_{\alpha=1}^{2} \frac{\vec{p}_\alpha^{\,2}}{2} + \sum_{\alpha=1}^{2} U(\vec{r}_\alpha) \right\} \Psi^{(0)} = E^{(0)}\Psi^{(0)}$$

If we omit spin components and use Bloch functions $(\frac{p^2}{2m} + V)\varphi_k = \varepsilon_k \varphi_k$ we can write

$$\sqrt{2}\Psi^{(0)} = \varphi_{k_1}(\vec{r}_1)\varphi_{k_2}(\vec{r}_2) + \varphi_{k_2}(\vec{r}_1)\varphi_{k_1}(\vec{r}_2), \qquad E^{(0)} = \varepsilon_{k_1} + \varepsilon_{k_2}, \qquad E^{(1)} = \int \Psi^{(0)^*} V(|\vec{r}_1 - \vec{r}_2|)\Psi^{(0)} d\vec{r}_1 d\vec{r}_2 .$$

At this point we just follow the standard perturbation theory.

However, Coulomb repulsion is not small. We can redefine interactions using specific properties of the system.

In a strong periodic potential electrons might be localized (Coulomb repelling might play an additional role as in Mott insulators). Electron density $\rho(\vec{r})$ is periodic or almost periodic. The exact Coulomb interaction can be approximated as $\int \frac{e\,dq_2}{|\vec{r_1} - \vec{r_2}|} = \int \frac{e\,\rho(\vec{r_2})d\vec{r_2}}{|\vec{r_1} - \vec{r_2}|}$; in the limit $V \to \infty$ this function is periodic having the same periodic properties as $\rho(\vec{r})$ has.

We can redefine now potentials in the Schrödinger's equation

$$H\Psi = \left\{ \sum_{\alpha=1}^{2} \frac{\vec{p}_\alpha^{\,2}}{2} + \sum_{\alpha=1}^{2} U(\vec{r}_\alpha) + V(|\vec{r}_1 - \vec{r}_2|) \right\} \Psi = E\Psi$$

where $U(\vec{r}_\alpha) \to U(\vec{r}_\alpha) + \frac{e}{2} \int \frac{\rho(\vec{r})d\vec{r}}{|\vec{r_\alpha} - \vec{r}|}$, and

$$V(|\vec{r}_1 - \vec{r}_2|) \to V(|\vec{r}_1 - \vec{r}_2|) - \frac{e}{2} \int \frac{\rho(\vec{r})d\vec{r}}{|\vec{r_1} - \vec{r}|} - \frac{e}{2} \int \frac{\rho(\vec{r})d\vec{r}}{|\vec{r_2} - \vec{r}|}.$$

The new "Coulomb interaction" should be small.

When the energy band is narrow a tight binding approximation gives the energy spectrum $\varepsilon_k = -t(\cos k_x + \cos k_y)$ (square 2D lattice; energy measures from the bottom).

We notice that for each $\vec{k} = (k_x, k_y)$ there is $\tilde{\vec{k}} = (\pi \operatorname{sgn}(k_x) - k_x, \pi \operatorname{sgn}(k_y) - k_y)$ so $\varepsilon_k + \varepsilon_{\tilde{k}} = 0$. *For this pair of momenta* $H_0\Psi^{(0)} = 0$. This make this particular pair of electrons special.

We should look at the application of perturbation theory to the whole system and see if forming $\vec{k}$-$\tilde{\vec{k}}$ pairs decreases the ground state energy.

Electrons with momenta $\vec{k}$ and $\tilde{\vec{k}}$ have the same group velocity. One possible interpretation is that when electrons are traveling "together" the exchange energy (which is negative for antiferromagnetics) gives the largest input into the energy of the system.

The further investigation might support the idea of the importance of the electron pairs with the same group velocity.

II.

Based on the idea of the importance of the electron pairs with the same group velocity it might be worthwhile to investigate properties of the system with Hamiltonian

$$H = \sum_{p\sigma}(\varepsilon_p - \mu)a^+_{p\sigma}a_{p\sigma} + U, \qquad \varepsilon_p = -t(\cos p_x + \cos p_y), \qquad U = -\frac{W}{N}V^+V, \qquad V^+ = \sum_{\substack{p \\ |\varepsilon_p|<|\varepsilon_F|}} a^+_{p\uparrow}a^+_{\tilde{p}\downarrow}$$

$$\vec{p} = (p_x, p_y), \quad \tilde{\vec{p}} = (\pi\,\mathrm{sgn}(p_x) - p_x, \pi\,\mathrm{sgn}(p_y) - p_y).$$

One of the approaches is using a Bogolyubov-like canonical transformation

$$a_{p\uparrow} = u_p b_{p\uparrow} - v_p b^+_{\tilde{p}\downarrow}, \qquad a_{p\downarrow} = u_p b_{p\downarrow} + v_p b^+_{\tilde{p}\uparrow}, \qquad u_p = u_{\tilde{p}} > 0, \qquad v_p = v_{\tilde{p}} > 0, \qquad u_p^2 + v_p^2 = 1.$$

III.

Another direction is applying to different Hamiltonians a BCS-like wave function

$$|E_0\rangle = \prod_{\substack{\vec{p} \\ |\varepsilon_p|<|\varepsilon_F|}} \{u_p + v_p a^+_{p\uparrow}a^+_{\tilde{p}\downarrow}\} \prod_{\substack{\vec{p},\sigma \\ \varepsilon_p<\varepsilon_F}} a^+_{p\sigma}|\mathrm{vac}\rangle.$$

IV.

One of the most important features of HTSC is having preformed electron pairs above the critical temperature. Preformed pairs might behave as bosons, and that points out at the Bose-gas as a possible source for the ideas. The ground state function of the Bose gas includes pairs of bosons with opposite momenta. In terms of electrons in HTSC that might be the sign of the importance of *the pairs of pairs* of electrons.

Another direction for research is applying to different Hamiltonians a wave function (here $\varepsilon_F < 0$)

$$|E_0\rangle = \prod_{\substack{\vec{p} \\ \varepsilon_F<\varepsilon_p<0}} \{u_p + v_p a^+_{p\uparrow}a^+_{\tilde{p}\downarrow} + v_p a^+_{-p\downarrow}a^+_{-\tilde{p}\uparrow} + w_p a^+_{p\uparrow}a^+_{\tilde{p}\downarrow}a^+_{-p\downarrow}a^+_{-\tilde{p}\uparrow}\} \prod_{\substack{\vec{p},\sigma \\ \varepsilon_p<\varepsilon_F}} a^+_{p\sigma}|\mathrm{vac}\rangle.$$

V.

When analyzing of the importance of *quartets* of electrons it might be interesting to develop a diagram technique with anomalous Green functions of the kind $<\Psi\Psi\Psi\Psi>$, which are *not* reducible to Gor'kov's Anomalous Green's function $<\Psi\Psi>$.

VI. When analyzing of the importance of *quartets* of electrons it might be interesting to develop an approach similar to Landau-Ginsburg approach, but with inclusion of the order parameter based on anomalous Green functions of the kind $<\Psi\Psi\Psi\Psi>$.